

# Unusual metallic conductivity of high- $T_c$ cuprates

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The intrinsic mechanisms of the unusual metallic transports of three types of relevant charge carriers (large polarons, excited (dissociated) polaronic components of bosonic Cooper pairs and bosonic Cooper pairs themselves) along the  $\text{CuO}_2$  layers of high- $T_c$  cuprates are identified and the new features of metallic conductivity in the  $\text{CuO}_2$  layers (i.e.  $ab$ -planes) of underdoped and optimally doped cuprates are explained. The in-plane conductivity of high- $T_c$  cuprates is associated with the metallic transports of such charge carriers at their scattering by lattice vibrations in thin  $\text{CuO}_2$  layers. The proposed charge transport theory in high- $T_c$  cuprates allows to explain consistently the distinctive features of metallic conductivity and the puzzling experimental data on the temperature dependences of their in-plane resistivity  $\rho_{ab}$ . In underdoped and optimally doped cuprates the linear temperature dependence of  $\rho_{ab}(T)$  above the pseudogap formation temperature  $T^*$  is associated with the scattering of polaronic carriers at acoustic and optical phonons, while the different (upward and downward) deviations from the linearity in  $\rho_{ab}(T)$  below  $T^*$  are caused by the pseudogap effect on the conductivity of the excited Fermi components of bosonic Cooper pairs and by the dominating conductivity of bosonic Cooper pairs themselves in the normal state of these high- $T_c$  materials.

**Keywords:** cuprate high-temperature superconductors, polarons, pseudogap effect, bosonic Cooper pairs, unusual metallic conductivity.

## Introduction

Underdoped and optimally doped copper-oxide (cuprate) high- $T_c$  superconductors exhibit distinctly different pseudogap features in the normal state, which cannot be explained within the framework of the existing theories based on the different non-standard models [1, 2]. The pseudogap features observed experimentally are manifested in the different electronic properties of these high- $T_c$

materials at temperatures lower than some characteristic temperature  $T^* > T_c$  (where  $T_c$  is the critical temperature of the superconducting transition). In particular, the pseudogap features are manifested in the temperature dependence of the unusual metallic conductivity of underdoped and optimally doped high- $T_c$  cuprates and in their temperature-dependent resistivity along  $\text{CuO}_2$  layers (i.e.  $ab$ -planes)  $\rho_{ab}$  at  $T < T^*$ , and various anomalous behaviors of  $\rho_{ab}(T)$  observed in these materials were not explained at all within the framework of different often discussed theoretical models (see [1-4] for a review). Most theoretical approaches are based on the extended Hubbard models (such as the resonating valence bond model [5], the marginal Fermi-liquid model [6], the nearly anti-ferromagnetic Fermi-liquid model [7], t-J models [2, 3]), which ignore without any justification the important electron-phonon interactions inherent high- $T_c$  cuprates and attempted to explain basically the linear dependence of  $\rho_{ab}(T)$  on temperature  $T$  above  $T^*$  and were not able to explain properly even such a metallic behavior of  $\rho_{ab}(T)$ . However, the extended Hubbard models are believed to be applicable for lightly doped cuprates, which are close to a half-filled case [3, 8].

As is well known, the existing experimental results indicate [9, 10] that strong electron-phonon interactions and polaronic effects are involved in the formation of a pseudogap and charge transport in high- $T_c$  cuprates. Actually, charge carriers being placed in a polar crystal will interact with the acoustic and optical phonons and their ground states are self-trapped (polaronic) states [11, 12]. Therefore, it is natural to assume that the Cooper pairing of doped electron or hole carriers can occur in the polaronic band and lead to the formation of unconventional Cooper pairs and the related BCS-like gap in the normal state of high- $T_c$  cuprates [13]. These polaronic Cooper pairs behave like bosons and the BCS-like gap appears at a temperature  $T^* > T_c$  in the normal state as a pseudogap. Below  $T^*$  polaronic (bosonic) Cooper pairs taking part in the metallic transport are scattered by acoustic and optical phonons.

The present paper is devoted to study the distinctive features of metallic conductivity in high- $T_c$  cuprates and the mechanisms of the metallic transports of polaronic carriers above  $T^*$  and two other types of charge carriers, such as excited (i.e. dissociated) Fermi components polaronic (bosonic) Cooper pairs and nondissociated bosonic Cooper pairs below  $T^*$  in these systems. It will be shown that the assumption of the existence of such relevant charge carriers taking part in the metallic transports in underdoped and optimally doped cuprates allows us to explain the new features of metallic conductivity in these high- $T_c$  materials and the puzzling experimental data on temperature dependence of  $\rho_{ab}(T)$ .

## Relevant charge carriers and their metallic transports along the $\text{CuO}_2$ layers

As is well known, hole carriers in ionic crystals (in particular, in alkali halides and doped cuprates) are self-trapped and would exist as polaronic carriers. Actually, experimental results give evidence about that the charge carriers in

doped cuprates are large polarons [11, 12]. Cooper pairing of these polarons may lead to the formation of incoherent bosonic Cooper pairs at some temperature  $T^*$  above their superconducting transition temperature  $T_c$  [13]. Therefore, we will consider scattering processes of large polarons and bosonic Cooper pairs by acoustic and optical phonons and determine the temperature dependences of the conductivity (resistivity) of doped high- $T_c$  cuprates both above  $T^*$  and below  $T^*$ . We show that the conductivity of underdoped and optimally doped cuprates along  $\text{CuO}_2$  layers above  $T_c$  is associated with the metallic transports of large polarons, excited Fermi components of polaronic Cooper pairs and very bosonic Cooper pairs. One can assume that polaronic carriers and bosonic Cooper pairs are scattered effectively by optical phonons having accordingly characteristic frequencies  $\omega_0 = \omega_{01}$  and  $\omega_0 = \omega_{02}$ . Above  $T^*$  the total relaxation time  $\tau_p(\varepsilon)$  of polaronic carriers with energy  $\varepsilon$  is determined from the expression [14]

$$\frac{1}{\tau_p(\varepsilon)} = \frac{1}{\tau_a(\varepsilon)} + \frac{1}{\tau_o}, \quad (1)$$

where  $\tau_a(\varepsilon) = A_p/t\sqrt{\varepsilon}$  is the relaxation time of large polarons scattered by acoustic phonons,  $\tau_o = B_p \exp[\hbar\omega_{01}/k_B T^*]$  is the relaxation time of these carriers scattered by optical phonons,  $A_p = \pi\hbar^2\rho_M v_s^2/\sqrt{2}E_d^2 m_p^{2/3} k_B T^*$ ,  $B_p = 4\sqrt{2}\pi\tilde{\varepsilon}(\hbar\omega_{01})^{3/2}/\omega_{01}^2 e^2 m_p^{1/2}$ ,  $t = T/T^*$ ,  $\rho_M$  is the mass density,  $v_s$  is the sound velocity,  $m_p$  is the polaron mass,  $E_d$  is the deformation potential,  $\tilde{\varepsilon} = \varepsilon_\infty/(1-\eta)$ ,  $\eta = \varepsilon_\infty/\varepsilon_0$ ,  $\varepsilon_\infty$  and  $\varepsilon_0$  are the high frequency and static dielectric constants, respectively.

We can take the components of the polaron mass  $m_{p1} = m_{p2} = m_{ab}$  and  $m_{p3} = m_c$  for the  $ab$ -plane and  $m_{p3} = m_c$  for the  $c$ -axis in the cuprates. Then the effective mass of polarons in the layered cuprates is given by  $m_p = (m_{ab}^2 m_c)^{1/3}$ .

If the electric field is applied in the  $x$ -direction (i.e. along the  $ab$ -plane), the conductivity of high- $T_c$  cuprates above  $T^*$  in the relaxation time approximation is defined as

$$\sigma_p(T > T^*) = -\frac{e^2}{4\pi^3} \int \tau_p(\varepsilon) v_x^2 \frac{\partial f_p}{\partial \varepsilon} d^3k, \quad (2)$$

where  $f_p(\varepsilon) = (e^{(\varepsilon-\mu)/k_B T} + 1)^{-1}$  is the Fermi distribution function,  $\varepsilon = \hbar^2 k^2/2m_p$  and  $v_x = \frac{1}{\hbar} \frac{\partial \varepsilon}{\partial k_x}$  are the energy and velocity of polarons.

In the case of an ellipsoidal energy surface, one can make the following transformations:  $k_x = m_{ab}^{1/2} k'_x$ ,  $k_y = m_{ab}^{1/2} k'_y$ ,  $k_z = m_c^{1/2} k'_z$ . Then the average kinetic energy of a polaron over the energy layer  $\Delta\varepsilon$  along three directions  $k_x$ ,  $k_y$  and  $k_z$  is the same and equal to one third of the total energy  $\varepsilon$ . Therefore, replacing  $v_x^2$  by  $\hbar^2 k_x'^2/m_{ab}$  and using the relation  $d^3k = (m_{ab}^2 m_c)^{1/2} d^3k'$ , we can write Eq.(2) in the form

$$\sigma_p(T > T^*) = -\frac{e^2}{4\pi^3} (m_{ab}^2 m_c)^{1/2} \int \tau_p(\varepsilon) \frac{\hbar^2 k_x'^2}{m_{ab}} \frac{\partial f_p}{\partial \varepsilon} d^3k'. \quad (3)$$

In Eq.(3) replacing  $k_x'^2$  by  $2\varepsilon/3\hbar^2$  and using further the carrier density  $n$  given by

$$n = \frac{2}{(2\pi)^3} \int f_p(k) d^3k = \frac{(2m_{ab}^2 m_c)^{1/2}}{\pi^2 \hbar^3} \int f_p(\varepsilon) \varepsilon^{1/2} d\varepsilon, \quad (4)$$

we obtain

$$\sigma_p(T > T^*) = \frac{2ne^2}{3m_{ab}} \frac{\int_0^\infty \tau_p(\varepsilon) \varepsilon^{3/2} (-\partial f_p / \partial \varepsilon) d\varepsilon}{\int_0^\infty f_p(\varepsilon) \varepsilon^{1/2} d\varepsilon}. \quad (5)$$

For a degenerate polaronic Fermi gas with the Fermi energy  $\varepsilon_F \gg k_B T$ , one can consider approximately that  $f_p(\varepsilon < \varepsilon_F) = 1$  and  $f_p(\varepsilon > \varepsilon_F) = 0$ . In this case the function  $-\partial f_p / \partial \varepsilon$  is nonzero only near  $\varepsilon_F$  and close to the  $\delta$ -function. Therefore, the integrals in Eq.(5) can be evaluated as

$$\int_0^\infty f_p(\varepsilon) \varepsilon^{1/2} d\varepsilon = \int_0^{\varepsilon_F} \varepsilon^{1/2} d\varepsilon = \frac{2}{3} \varepsilon_F^{3/2}, \quad (6)$$

$$\begin{aligned} \int_0^\infty \tau_p(\varepsilon) \varepsilon^{3/2} \left(-\frac{\partial f_p}{\partial \varepsilon}\right) d\varepsilon &= B_p e^{\alpha_p/t} \int_0^\infty \frac{\varepsilon^{3/2}}{1 + c_p(t) \sqrt{\varepsilon}} \delta(\varepsilon - \varepsilon_F) d\varepsilon = \\ &= B_p e^{\alpha_p/t} \frac{\varepsilon_F^{3/2}}{1 + c_p(t) \sqrt{\varepsilon}}, \end{aligned} \quad (7)$$

where  $\alpha_p = \hbar \omega_{01} / k_B T^*$ ,  $c_p(t) = (B_p / A_p) t e^{\alpha_p/t}$ .

Substituting Eqs.(6) and (7) into Eq.(5), we obtain the expression for the conductivity of high- $T_c$  cuprates above  $T^*$  along  $\text{CuO}_2$  layers

$$\sigma_{ab}(t > 1) = \sigma_p(t > 1) = \frac{ne^2 B_p e^{\alpha_p/t}}{m_{ab} [1 + c_p(t) \sqrt{\varepsilon}]}. \quad (8)$$

Below  $T^*$  the polaronic carriers in the energy layer of width  $\varepsilon_A$  near the Fermi surface take part in the BCS-like Cooper pairing and form polaronic (bosonic) Cooper pairs. The number of such polaronic carriers is determined from the relation

$$n_p^* + 2n_B = 2 \sum_k [u_k^2 f_c(k) + v_k^2 (1 - f_c(k))], \quad (9)$$

where  $n_p^* = 2 \sum_k u_k^2 f_c(k)$  is the number of the excited Fermi components of Cooper pairs,  $n_B = \sum_k v_k^2 [1 - f_c(k)]$  is the number of bosonic Cooper pairs,  $f_c(k) = f(E(k)) = [e^{E(k)/k_B T} + 1]^{-1}$ ,  $E(k) = \sqrt{\xi^2(k) + \Delta_F^2}$ ,  $u_k = \sqrt{\frac{1}{2} [1 + \xi(k)/E(k)]}$ ,  $v_k = \sqrt{\frac{1}{2} [1 - \xi(k)/E(k)]}$ ,  $\xi(k) = \varepsilon(k) - \varepsilon_F$ .

The contribution of the excited Fermi components of Cooper pairs to the conductivity of high- $T_c$  cuprates below  $T^*$  in the relaxation time approximation is defined as

$$\sigma_p^*(T < T^*) = -\frac{e^2}{8\pi^3} \int \tau_{BCS}(\xi) v_x^2 \frac{\xi}{E} \left(1 + \frac{\xi}{E}\right) \frac{\partial f_c}{\partial E} d^3k. \quad (10)$$

If we consider a thin  $\text{CuO}_2$  layer of the cuprate superconductor with an ellipsoidal energy surface, the expression for  $\sigma_p^*(T < T^*)$  can be written as

$$\sigma_p^*(T < T^*) = \frac{ne^2}{3m_{ab}} \frac{\int_{-\varepsilon_A}^{\varepsilon_A} \tau_{BCS}(\xi + \mu) (\xi + \varepsilon_F)^{3/2} \frac{\xi}{E} \left(1 + \frac{\xi}{E}\right) \left(-\frac{\partial f_c}{\partial E}\right) d\xi}{\int_0^\infty f_p(\varepsilon) \varepsilon^{1/2} d\varepsilon}. \quad (11)$$

If we use the property of  $\delta$ -function  $\delta[E(k') - E(k)] = (d\varepsilon/dE)\delta[\varepsilon(k') - \varepsilon(k)]$  in the expression for the scattering probability of the Fermi components of Cooper pairs below  $T^*$ , the relaxation time of polaronic components of Cooper pairs is determined from the relation [15]

$$\tau_{BCS}(\xi + \varepsilon_F) = \frac{E}{|\xi|} \tau_p(\xi + \varepsilon_F). \quad (12)$$

Substituting Eq.(12) into Eq.(11), we obtain

$$\sigma_p^*(t < 1) = \frac{ne^2}{3m_{ab}} \frac{\int_{-\varepsilon_A}^{\varepsilon_A} \tau_p(\xi + \mu)(\xi + \varepsilon_F)^{3/2} \frac{\xi}{|\xi|} (1 + \frac{\xi}{E}) (-\frac{\partial f_\varepsilon}{\partial E}) d\xi}{\int_0^\infty f_p(\varepsilon) \varepsilon^{1/2} d\varepsilon}. \quad (13)$$

The BCS-like energy gap  $\Delta_F$  (which exists above  $T_c$  as a pseudogap) and characteristic temperature  $T^*$  at which  $\Delta_F$  becomes zero is determined from the BCS-like gap equation [13]

$$\frac{1}{\lambda_p^*} = \int_0^{\varepsilon_A} \frac{d\xi}{\sqrt{\xi^2 + \Delta_F^2(T)}} \text{th} \frac{\sqrt{\xi^2 + \Delta_F^2(T)}}{2k_B T}, \quad (14)$$

where  $\lambda_p^*$  is the BCS-like coupling constant.

We now consider the contribution of bosonic Cooper pairs to the in-plane conductivity of the cuprates above  $T_c$  and define the mass of such a Cooper pair as  $m_B = (M_{ab}^2 M_c)^{1/3}$ , where  $M_{ab} = 2m_{ab}$  and  $M_c = 2m_c$  are the in-plane and out-of-plane ( $c$ -axis) masses of the polaronic Cooper pairs, respectively. The density of bosonic Cooper pairs below  $T^*$  is determined from the relation

$$n_B = \frac{(m_{ab}^2 m_c)^{1/2}}{2\sqrt{2}\pi^2 \hbar^3} \int_{-\varepsilon_A}^{\varepsilon_A} [1 - \frac{\xi}{E}] (\xi + \varepsilon_F)^{1/2} \frac{e^{E/k_B T}}{e^{E/k_B T} + 1} d\xi, \quad (15)$$

Numerical calculations of the concentration  $n_B$  of Cooper pairs and the temperature  $T_{BEC}$  of their Bose-Einstein condensation (BEC) show that just below  $T^*$  the value of  $T_{BEC}$  is very close  $T^*$ , but somewhat below  $T^*$ ,  $T_{BEC} \gg T^*$ . Therefore, polaronic Cooper pairs below  $T^*$  can be considered as an ideal Bose gas with the chemical potential  $\mu_B = 0$ . Below  $T_{BEC}$ , the density of bosonic Cooper pairs with zero and non-zero energies  $\varepsilon$  is given by

$$n_B = n_B(\varepsilon > 0) + n_B(\varepsilon = 0). \quad (16)$$

where

$$n_B(\varepsilon > 0) = \frac{(M_{ab}^2 M_c)^{1/2}}{\sqrt{2}\pi^2 \hbar^3} \int_0^\infty \frac{\varepsilon^{1/2} d\varepsilon}{e^{E/k_B T} - 1} = n_B \left( \frac{T}{T_{BEC}} \right)^{3/2}. \quad (17)$$

Clearly, only the Cooper pairs with  $K \neq 0$  and density  $n_B(\varepsilon > 0)$  contribute to the conductivity of high- $T_c$  cuprates below  $T^*$ , so that the conductivity of bosonic Cooper pairs below  $T^*$  in the relaxation time approximation is defined as

$$\sigma_B(T < T^*) = -\frac{e^2}{2\pi^3} \int_0^\infty \tau_B(\varepsilon) v_x^2 \frac{\partial f_B}{\partial \varepsilon} d^3 K. \quad (18)$$

where is the relaxation time  $\tau_B(\varepsilon)$  of bosonic Cooper pairs at their scattering by acoustic and optical phonons is defined as  $\tau_B(\varepsilon) = \tau_a^c(\varepsilon)\tau_o^c/(\tau_a^c + \tau_o^c)$ ,  $\tau_a^c(\varepsilon) = A_c/t\sqrt{\varepsilon}$ ,  $A_c = \pi\hbar^4\rho_M v_s^2/\sqrt{2}E_d^2 m_B^3/2k_B T^*$ ,  $\tau_o^c = B_c e^{\hbar\omega_{02}/k_B T^*}$ ,  $B_c = \sqrt{2}\pi(\hbar\omega_{02})^{3/2}/\omega_{02}^2 e^2 \sqrt{m_B}$ ,  $f_B(\varepsilon) = (e^{\varepsilon/k_B T} - 1)^{-1}$  is the Bose distribution function.

Here we make the transformation  $K = M_\alpha^{1/2} K'$  (where  $\alpha = x, y, z$ ) analogously as in the case of polaronic carriers. In the case of the ellipsoidal energy surface, the expression for the conductivity  $\sigma_B(T < T^*)$  of bosonic Cooper pairs at their scattering by acoustic and optical phonons in the  $\text{CuO}_2$  layers of the cuprates may be written in the form

$$\sigma_B(T < T^*) = \frac{e^2}{2\pi^3} (M_{ab}^2 M_c)^{1/2} \int_0^\infty \tau_B(\varepsilon) \frac{\hbar^2 K_\alpha'^2}{M_\alpha} \left(-\frac{\partial f_B}{\partial \varepsilon}\right) d^3 K'. \quad (19)$$

Using the relation  $K_\alpha'^2 = 2\varepsilon/3\hbar^2$  and Eq.(17) and after replacing  $M_\alpha$  in Eq.(19) by  $M_{ab}$ , the expression for  $\sigma_B(T < T^*)$  can be written as

$$\sigma_B(T < T^*) = \frac{8n_B(T/T_{BEC})^{3/2} e^2}{3M_{ab}} \frac{\int_0^\infty \tau_B(\varepsilon) \varepsilon^{3/2} \left(-\frac{\partial f_B}{\partial \varepsilon}\right) d\varepsilon}{\int_0^\infty f_B(\varepsilon) \varepsilon^{1/2} d\varepsilon}, \quad (20)$$

where  $T_{BEC} = 3.31\hbar^2 n_B^{2/3}/k_B m_B$ .

After performing to integration in the denominator of this expression,  $\sigma_B(T < T^*)$  may be written in the form

$$\begin{aligned} \sigma_B(T < T^*) &= 0,19 \frac{m_B^{3/2} e^2}{M_{ab} \hbar^3} \int_0^\infty \tau_B(\varepsilon) \varepsilon^{3/2} \left(-\frac{\partial f_B}{\partial \varepsilon}\right) d\varepsilon = \\ &= 0,19 \frac{m_B^{3/2} e^2}{M_{ab} \hbar^3} \frac{B_c e^{\alpha_c/t}}{k_B T^* t} \int_0^\infty \frac{\varepsilon^{3/2} e^{\varepsilon/k_B T^* t}}{(e^{\varepsilon/k_B T^* t} - 1)^2 (1 + \beta_c(t) \sqrt{\varepsilon})} d\varepsilon, \end{aligned} \quad (21)$$

where  $\beta_c(t) = B_c t e^{\alpha_c/t}/A_c$ ,  $\alpha_c = \hbar\omega_{02}/k_B T^*$ .

The total conductivity of the excited polaronic component of Cooper pairs and the bosonic Cooper pairs below  $T^*$  along  $\text{CuO}_2$  layers in high- $T_c$  cuprates is determined from the expression

$$\sigma_{ab}(t < 1) = \sigma_p^*(t < 1) + \sigma_B(t < 1). \quad (22)$$

We now demonstrate that the above theory of metallic conductivity in the normal state of high- $T_c$  cuprates along  $\text{CuO}_2$  layers adequately and fairly well describes unusual temperature dependences of  $\rho_{ab}(T)$  above and below  $T^*$  observed in the underdoped regime. Our results are compared with the well-established experimental data for  $\rho_{ab}(T)$ , obtained in various underdoped high- $T_c$  materials, with the use of the realistic set of their parameters. In so doing, the resistivity of high- $T_c$  cuprates along  $\text{CuO}_2$  layers  $T > T^*$  and  $T < T^*$  are determined from the expression  $\rho_{ab}(T > T^*) = \rho_0 + 1/\sigma_{ab}(T > T^*)$  and  $\rho_{ab}(T < T^*) = \rho_0 + 1/\sigma_{ab}(T < T^*)$ , respectively, where  $\rho_0$  is the residual resistivity. These expressions for  $\rho_{ab}(T)$  allow us to obtain experimental curves of the temperature dependences of  $\rho_{ab}(T)$  in various high- $T_c$  cuprates both above  $T^*$  and below

$T^*$  with the use of their characteristic parameters. In Fig.1 the calculated and experimentally measured temperature dependences of the resistivity  $\rho_{ab}(T)$  are presented for the underdoped high- $T_c$  superconductor  $Y_{1-z}Pr_zBa_2Cu_3O_{7-\delta}$  (with  $z = 0.23$ ,  $\delta > 0.5$  and  $T_c \simeq 67K$  [16]). As can be seen in Fig.1, the resistivity  $\rho_{ab}(T)$  shows the linear  $T$  dependence above  $T^*$  and starts to deviate downward from the  $T$ -linear behavior below  $T^*$ . We see that in the high- $T_c$  cuprate  $Y_{1-z}Pr_zBa_2Cu_3O_{7-\delta}$ ,  $\rho_{ab}(T)$  deviates downward from the linear law below the characteristic temperature  $T^* = 150$  K (below which the BCS-like pseudogap  $\Delta_F$  appears on the Fermi surface at  $\lambda_p^* = 0.55$ ) and it is associated with the dominating contribution of the conductivity of incoherent bosonic Cooper pairs to  $\sigma_{ab}(T < T^*)$ . In order to reach good agreement between theory and experiment, we have used the following set of the characteristic parameters of the underdoped high- $T_c$  system  $Y_{1-z}Pr_zBa_2Cu_3O_{7-\delta}$  in our numerical calculations:  $v_s = 5.9 \cdot 10^5$  cm/s,  $\rho_M = 6$  g/cm<sup>3</sup>,  $m_{ab} = 3.003 \cdot 10^{-27}$  g,  $\tilde{\epsilon} = 5.1$ ,  $m_p = 3.512 \cdot 10^{-27}$  g,  $n = 1.31 \cdot 10^{21}$  cm<sup>-3</sup>,  $\hbar\omega_{01} = 0.057$  eV,  $\hbar\omega_{02} = 0.07$  eV and  $\rho_0 = 0.08$  mΩcm. The results of the numerical calculations of  $\rho_{ab}(T)$  for other underdoped high- $T_c$  material  $YBa_2(Cu_{1-x}Zn_x)_3O_{7-\delta}$  (with  $x = 0.02$ ,  $\delta = 0.28$  and  $T_c = 30$  K) are also compared with experimental data for  $\rho_{ab}(T)$ , presented in Ref.[17] for this high- $T_c$  superconductor (see Fig.2). In high- $T_c$  cuprate superconductor  $YBa_2(Cu_{1-x}Zn_x)_3O_{7-\delta}$  the other tendency in the behavior of  $\rho_{ab}(T)$  is observed below  $T^*$ , i.e.,  $\rho_{ab}(T)$  deviates upward from the linear  $T$ -dependence below  $T^*$ . In order to reach reasonable agreement between theoretical and experimental values of  $\rho_{ab}(T)$  for such a system, we have used the following set of the characteristic parameters of the underdoped cuprate material  $YBa_2(Cu_{1-x}Zn_x)_3O_{7-\delta}$  in our numerical calculations:  $v_s = 4.5 \cdot 10^5$  cm/s,  $\rho_M = 5$  g/cm<sup>3</sup>,  $m_{ab} = 2.355 \cdot 10^{-27}$  g,  $\tilde{\epsilon} = 5.8$ ,  $m_p = 2.604 \cdot 10^{-27}$  g,  $n = 0.55 \cdot 10^{21}$  cm<sup>-3</sup>,  $\hbar\omega_{01} = 0.047$  eV,  $\hbar\omega_{02} = 0.046$  eV and  $\rho_0 = 0.4$  mΩcm. As can be seen in Fig.2, the temperature-dependent resistivity  $\rho_{ab}(T)$  deviates upward from the linear law with decreasing the temperature below  $T^* = 125$  K (determined from Eq.(14) at  $\lambda_p^* = 0.52$ ). Such a behavior of  $\rho_{ab}(T)$  is associated with the pseudogap effect on the conductivity of the excited (dissociated) polaronic components of bosonic Cooper pairs, which gives significant contribution to the resulting conductivity  $\sigma_{ab}(t < 1)$  and it is comparable with the conductivity of bosonic Cooper pairs themselves.

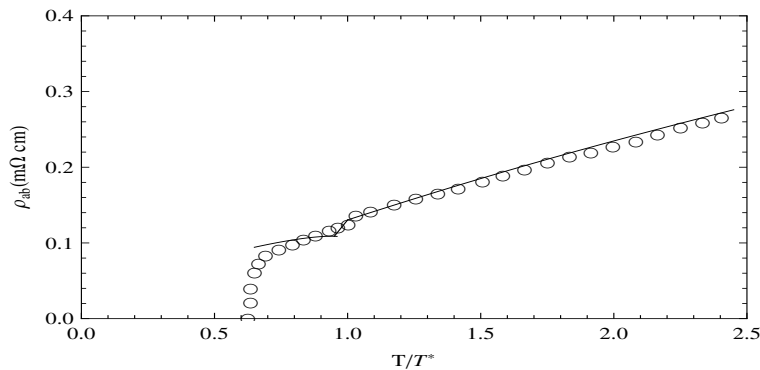


Figure 1. Comparison of the calculated results for  $\rho_{ab}(T)$  (solid line) with the experimental data on the temperature dependences of  $\rho_{ab}$  above and below  $T^*$  for  $Y_{1-z}Pr_zBa_2Cu_3O_{7-\delta}$  open circles [16].

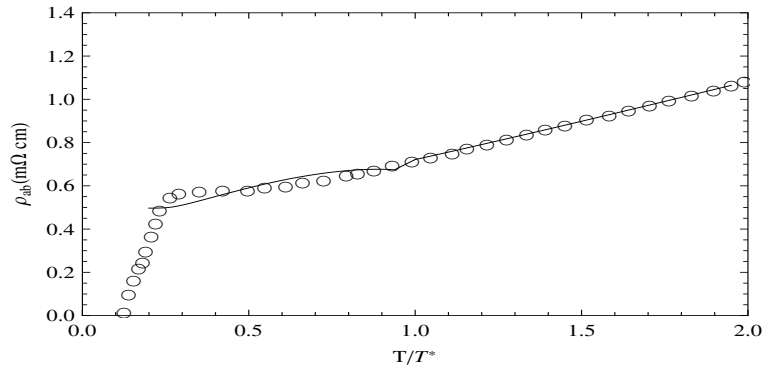


Figure 2. Comparison of the calculated results for  $\rho_{ab}(T)$  (solid line) with the experimental data on the temperature dependences of  $\rho_{ab}$  above and below  $T^*$  for  $\text{YBa}_2(\text{Cu}_{1-z}\text{Zn}_x)_3\text{O}_{7-\delta}$  open circles [17].

## Conclusion

Thus, we have identified the intrinsic mechanisms of the charge transports in the normal state of underdoped and optimally doped high- $T_c$  cuprates along the  $\text{CuO}_2$  layers by considering the important contributions of three types of charge carriers (large polarons, excited polaronic components of bosonic Cooper pairs and bosonic Cooper pairs themselves) to the unusual metallic conductivity in these high- $T_c$  materials. In so doing, we have explained the distinctive features of metallic conductivity in high- $T_c$  cuprates and the puzzling experimental data on the temperature dependences of their in-plane resistivity  $\rho_{ab}$ . In underdoped and optimally doped cuprates the relevant charge carriers above  $T^*$  are large polarons, while two other types of relevant charge carriers, such as the excited Fermi components of bosonic Cooper pairs and nondissociated bosonic Cooper pairs exist below  $T^*$ . We have shown that in these materials the scattering of polaronic carriers at acoustic and optical phonons leads to the linear  $T$ -dependence of  $\rho_{ab}(T)$  above  $T^*$ . Further, we have found that the BCS-like pseudogap effect and the excessive conductivity of bosonic Cooper pairs in the normal state of high- $T_c$  cuprates are responsible for the pronounced nonlinear temperature dependence of  $\rho_{ab}(T)$  and for different downward (at  $\omega_{01} > \omega_{02}$ ) and upward (at  $\omega_{01} < \omega_{02}$ ) deviations from the  $T$ -linear behavior in  $\rho_{ab}(T)$  below  $T^*$ . Downward and upward deviations of  $\rho_{ab}(T)$  from the  $T$ -linear law below  $T^*$  are associated with the competing contributions (coming from the conductivity of excited Fermi components of bosonic Cooper pairs and the conductivity of nondissociated bosonic Cooper pairs) to the resulting conductivity  $\sigma_{ab}(t < 1)$  of high- $T_c$  cuprates.

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