

# Calculation of proton transfer cross sections in the $^{14}\text{N} + ^{12}\text{C}$ reaction at 116 MeV using the DWBA method

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The proton transfer in the reaction  $^{12}\text{C}(^{14}\text{N},^{13}\text{C})^{13}\text{N}$  are discussed. Calculations of level energies in the shell model of deformed and spherical nuclei are performed for  $^{14}\text{N}$  nuclei. The present theoretical analysis of the  $^{12}\text{C}(^{14}\text{N},^{13}\text{C})^{13}\text{N}$  reaction at 116 MeV was performed by means of the FRESCO code. Our theoretically calculated differential cross sections give a fair description of the experimental data for the proton transfer reaction. Based on calculation for 116 MeV predicted calculations for region from Coulomb barrier to a maximum energy available at the DC-60 heavy ion accelerator were made.

**Keywords:** DWBA method, shell model, deformed and spherical nuclei, proton transfer.

## Introduction

Direct nuclear reactions can give information about reaction mechanisms namely the formation of a compound nucleus and its subsequent removal of excitation by the evaporation of particles. The second reaction mechanism is the transfer of a nucleon from one nucleus to another during a grazing collision [1-3]. This article is devoted to the study of the reaction mechanism of  $^{12}\text{C}(^{14}\text{N},^{13}\text{C})^{13}\text{N}$  at 116 MeV and near barrier energies. Experimental data [4] on the proton transfer

reaction  $^{12}\text{C}(^{14}\text{N},^{13}\text{C})^{13}\text{N}$  at 116 MeV were used to determine the real and imaginary parts of effective optical potential and the spectroscopic amplitudes. Based on this calculation angular distributions for the reaction  $^{12}\text{C}(^{14}\text{N},^{13}\text{C})^{13}\text{N}$  at near barrier energies were predicted. It can be used for further experiments on proton transfer reaction at the DC-60 heavy ion accelerator (Nur-Sultan, Kazakhstan).

The experiments at the DC-60 heavy ion accelerator (Nur-Sultan, Kazakhstan) in nuclear physics are focused, first of all, on obtaining the missing information on the internuclear interaction potentials at large distances - on the periphery of the nucleus. The cross sections of nuclear reactions occurring both in a high-temperature plasma and in stars strongly depend on the potential values in this region. Main parameters of the accelerated ions beams are presented on Table 1 [5].

Table 1.  
Main parameters of the accelerated ions beams [5].

Type of ions	Li – Xe
A/Z	6 – 12
Energy of the accelerated ions	0.4 – 1.75 MeV/nucleon
Ions energy scattering	2 %
Discrete changes in ion energy	by changing the ionic charge (A/Z ratio)
Smooth variation of the ion energy	30% (+/-15%) due to variations in the magnetic field

The maximum energy of the projectile nucleus in the laboratory system is

$$E_{\text{lab.max}} = 1.75A_1 \text{ (MeV)}. \tag{1}$$

It corresponds to the maximum energy of nuclei in the center of mass system

$$E_{\text{c.m.max}}(A_1, A_2) = 1.75A_1 \frac{1}{1 + \frac{A_1}{A_2}} = 1.75 \frac{A_1 A_2}{A_1 + A_2} \text{ (MeV)}. \tag{2}$$

Here,  $A_1$  and  $A_2$  are the masses of colliding nuclei. For the  $^{14}\text{N} + ^{12}\text{C}$  the maximal energy available at the DC-60 heavy ion accelerator [5] is 24.5 MeV ( $E_{\text{c.m.max}} = 11.31 \text{ MeV}$ ).

To estimate the height of the Coulomb barrier  $V_B$ , we can use the value of the Coulomb energy of nuclei in contact with surfaces at a distance between the centers of nuclei

$$V_B(A_1, A_2) = 1.44 \frac{Z_1 Z_2}{R_{\text{cont}}(A_1, A_2)} 0.76 \text{ (MeV)}. \tag{3}$$

where  $R_{\text{cont}} = 1.25 (A_1^{1/3} + A_2^{1/3}) \text{ fm}$ ,  $Z_1$  and  $Z_2$  are the charges of colliding nuclei.

For the reaction  $^{14}\text{N} + ^{12}\text{C}$   $V_B = 7.89 \text{ MeV}$ . This value of  $V_B$  corresponds to Coulomb barrier calculated by using the parametrization [6] for the nucleus-nucleus potential.

## DWBA calculations

The present theoretical analysis of the  $^{12}\text{C}(^{14}\text{N}, ^{13}\text{C})^{13}\text{N}$  reaction at the projectile energy of 116 MeV was performed by means of the FRESKO code [7,8]. Since the  $^{14}\text{N}$  nucleus is deformed, we have employed the value of  $\beta_2 = -0.15$  [9] (for the quadrupole-deformation parameter) to calculate neutron single-particle levels. The proton-separation energy for the  $^{14}\text{N}$  nucleus is 7.55 MeV, and its spin-parity is  $1+$ . Proton levels in the shell model of a deformed and a spherical nucleus are shown in Figure 1. Calculations for the nuclear shell model were performed by using NRV [10, 11]. Also, potential parameters for model are given in Figure 1. As seen in the Figure 1, the occupied proton levels of the  $^{14}\text{N}$  nucleus in the shell model of a deformed and a spherical nucleus have fairly close structures. In the calculations below, to describe the structure of the  $^{14}\text{N}$ , we use the shell model of the spherical nucleus.

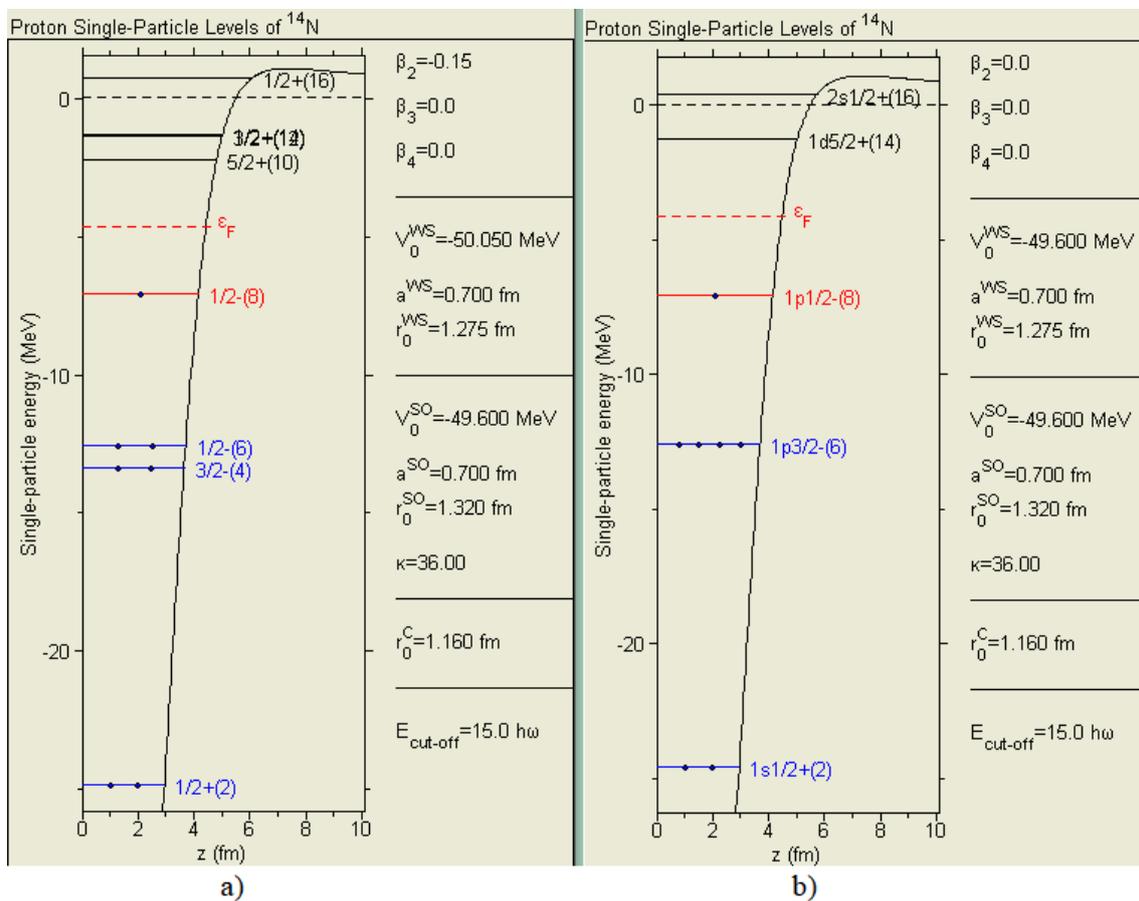


Figure 1. Proton levels in the model of a deformed (a) and a spherical (b) nucleus for  $^{14}\text{N}$  calculated by using NRV [10, 11].

By employing the shell model of the spherical nucleus to describe the structure of nuclei, we have calculated the differential cross sections for transfer of proton from the  $1p_{1/2}$  level in the  $^{14}\text{N}$  nucleus to the free  $1p_{1/2}$  level in the  $^{12}\text{C}$  nucleus. The proton-separation energy for the  $^{14}\text{N}$  nucleus is 1.944 MeV. All calculations were performed in the Born approximation (BA) with distorted waves (DW) (DWBA) method implemented in the FRESKO code [7, 8]. The Woods-Saxon

optical potential

$$V(r) = \frac{V_0}{1 + \exp\left(\frac{r-r_0(A_1^{1/3}+A_2^{1/3})}{a_0}\right)} + i \frac{W_0}{1 + \exp\left(\frac{r-r_W(A_1^{1/3}+A_2^{1/3})}{a_W}\right)} \quad (4)$$

was used to construct a precise quantum-mechanical description of the relative motion of participant nuclei in the entrance and exit reaction channels. The potential parameters (depths of the real  $V_0$  and imaginary  $W_0$  parts of the optical potential) were varied to obtain a satisfactory description of experimental data. The values of geometric parameters of the potential were fixed and calculated by parameterization [6]. The radius of the Coulomb potential was set to the value of  $r_C = 1.3$  fm. The optical potential parameters for the entrance and exit reaction channels are given in Table 2.

The entrance- and exit-channel potentials were used for calculation of the distorted waves, whereas the mechanism of the  $A + b \rightarrow a + B$  ( $A = a + v$ ,  $B = b + v$ ) reaction was determined by the transferred-nucleon and/or transferred-cluster wave function [12]:

$$T_{\text{prior}} = \int dR_\alpha dR_\beta \chi_\beta^{(-)}(R_\beta) I_{\beta\alpha}(R_\beta, R_\alpha) \chi_\alpha^{(+)}(R_\alpha), \quad (5)$$

$$I_{\beta\alpha}(R_\beta, R_\alpha) = (\phi_a \phi_B | V_{vb} + U_{ab} - U_\alpha | \phi_A \phi_b). \quad (6)$$

Here, the internal wave functions for the initial ( $\phi_A \phi_b$ ) and final ( $\phi_a \phi_B$ ) nuclei in Equation (6) play an important role in determining the transmission amplitude. To determine the internal wave functions, it is necessary to know: the spin parity  $J^\pi$  of the state of the residual and "composite" nucleus, the angular momentum  $l$  of the transferred particle relative to the core "core" and the number of nodes  $N$  in the radial wave function. The internal wave function with fixed quantum numbers is found by varying the depth of the Woods-Saxon potential with a fixed "geometry" (radius and diffuseness) until the equality between the particle separation energy and the energy of the corresponding particle level taken with the opposite sign is achieved.

Table 2.

Main parameters of the accelerated ions beams [5].

Reaction channel	$V_0$ , MeV	$r_0$ , fm	$a_0$ , fm	$W_0$ , MeV	$r_W$ , fm	$a_W$ , fm
$^{14}\text{N} + ^{12}\text{C}$	44.354	1.161	0.589	10.788	1.161	0.589
$^{13}\text{C} + ^{13}\text{N}$	28.832	1.161	0.589	5.708	1.161	0.589

Spectroscopic amplitudes for proton transfer are free calculation parameters. Spectroscopic amplitudes for overlaps  $\langle ^{14}\text{N} | ^{13}\text{C} \rangle_{\text{g.s.}}$  and  $\langle ^{12}\text{C} | ^{13}\text{N} \rangle_{\text{g.s.}}$  were 0.8 and 1.0, respectively. These parameters were chosen to achieve good agreement of the experimental data and theoretical values of the cross sections. The results of theoretical calculations of cross sections for proton transfer in the  $^{12}\text{C}(^{14}\text{N}, ^{13}\text{C})^{13}\text{N}$  reaction at energy of 116 MeV are shown in Figure 2. Our theoretically calculated

differential cross sections give a fair description of the experimental data for the proton transfer reaction. The probability of the formation of a compound nucleus and its subsequent removal of excitation by the evaporation of particles due to reactions is small. Since the angular distribution during the decay of a compound nucleus is almost isotropic; therefore, the deep minimum and peak observed in the angular distributions (Figure 2) for the ground state of  $^{13}\text{C}$  indicate a small contribution of compound nucleus formation.

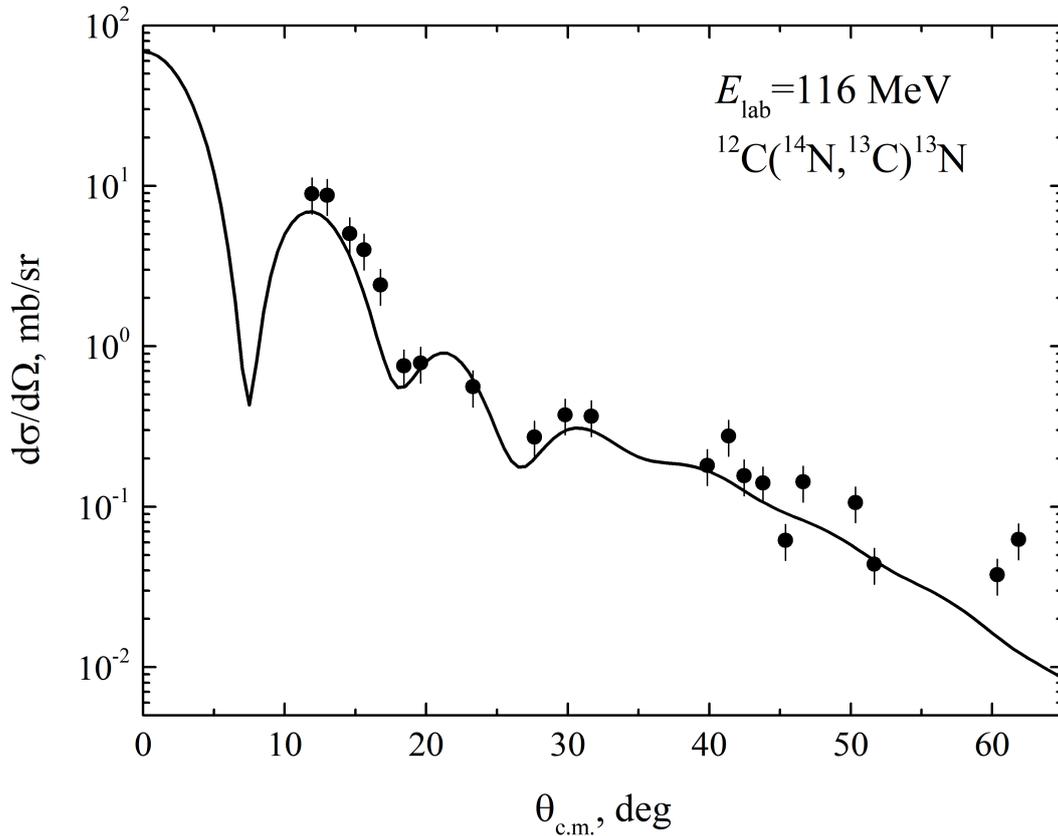


Figure 2. Differential cross section for the proton transfer in the  $^{12}\text{C}(^{14}\text{N}, ^{13}\text{C})^{13}\text{N}$  reaction at an energy of 116 MeV: (closed squares) experimental data and (solid curve) results of calculations using the DWBA method.

Based on calculation the angular distributions for the reaction  $^{12}\text{C}(^{14}\text{N}, ^{13}\text{C})^{13}\text{N}$  at 116 MeV predicted calculations for region from Coulomb barrier  $V_B$  to were made ( $V_B = 7.89$  MeV,  $E_{c.m.max} = 11.31$  MeV). Chosen energies in center mass system  $E_{c.m.}$  were 8.0, 9.0, 10.0 and 11.31 MeV (in the laboratory system  $E_{lab}$  are 17.33, 19.5, 21.67 and 24.5 MeV, respectively). The results of theoretical calculations of cross sections for proton transfer in the  $^{12}\text{C}(^{14}\text{N}, ^{13}\text{C})^{13}\text{N}$  reaction at chosen energy are shown in Figure 3. Calculation parameters for the reaction at energy of 116 MeV were used to calculate the transfer reaction cross sections at chosen energies. But nevertheless, for a more accurate determination of the parameters, it is necessary to conduct an experiment.

There are very few experimental data on proton transfer in the  $^{14}\text{N} + ^{12}\text{C}$  reaction. After analyzing the experimental data [4] in the framework of DWBA method, the total proton transfer cross sections were calculated (Figure 4). The value of the calculated total cross section in the framework of DWBA (at an energy of  $E_{lab} = 116$  MeV) is 5.568 mbarn. The value of experimental cross section in

Figure 4 was calculated by integration of approximated function for experimental data on differential cross section for the proton transfer at 116 MeV ( $E_{c.m.} = 53.5$  MeV).

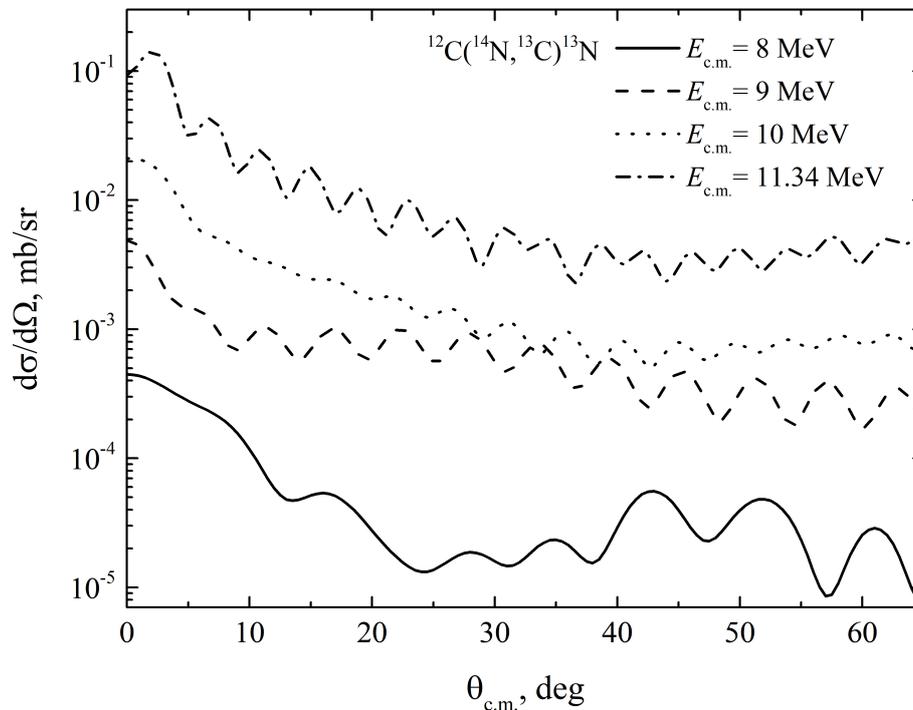


Figure 3. Differential cross section for the proton transfer in the  $^{12}\text{C}(^{14}\text{N}, ^{13}\text{C})^{13}\text{N}$  reaction at near barrier energies calculated by using the DWBA method.

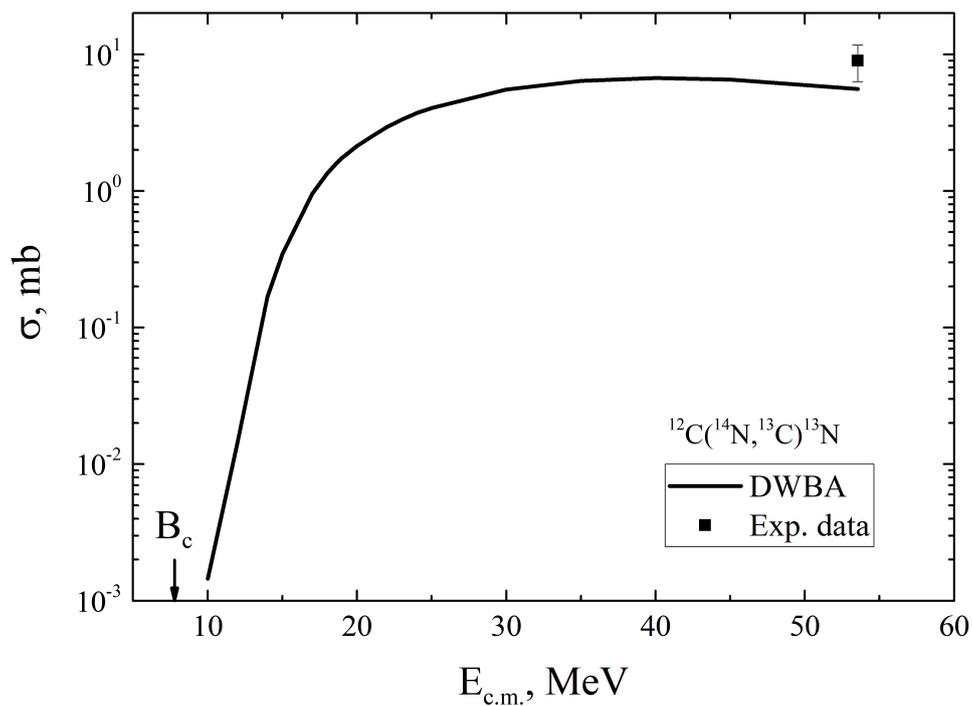


Figure 4. Cross sections for the  $^{12}\text{C}(^{14}\text{N}, ^{13}\text{C})^{13}\text{N}$  reaction.

## Conclusion

The differential cross sections for transfer of proton from the  $1p_{1/2}$  level in the  $^{14}\text{N}$  nucleus to the free  $1p_{1/2}$  level in the  $^{12}\text{C}$  nucleus have been calculated. The present theoretical analysis of the  $^{12}\text{C}(^{14}\text{N},^{13}\text{C})^{13}\text{N}$  reaction at the projectile energy of 116 MeV was performed by means of the FRESKO code. Our theoretically calculated differential cross sections give a fair description of the experimental data for the proton transfer reaction. Based on calculation the angular distributions for the reaction  $^{12}\text{C}(^{14}\text{N},^{13}\text{C})^{13}\text{N}$  at 116 MeV predicted calculations for region from Coulomb barrier  $V_B$  to  $E_{c.m.max}$  were made. Our results can be used for prepare experiment on proton transfer reaction at the DC-60 heavy ion accelerator (Nur-Sultan, Kazakhstan).

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